The Meissner Effect and Vortex Expulsion in Color-Superconducting Quark stars, and its Role for Re-heating of Magnetars

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Compact stars made of quark matter rather than confined hadronic matter, are expected to form a color superconductor. This superconductor ought to be threaded with rotational vortex lines, within which the star's interior magnetic field is at least partially confined. The vortices (and thus magnetic flux) would be expelled from the star during stellar spin-down, leading to magnetic reconnection at the surface of the star and the prolific production of thermal energy. In this paper, we show that this energy release can re-heat quark stars to exceptionally high temperatures, such as observed for Soft Gamma Repeaters (SGRs), Anomalous X-Ray pulsars (AXPs), and X-ray dim isolated neutron stars (XDINs). Moreover, our numerical investigations of the temperature evolution, spin-down rate, and magnetic field behavior of such superconducting quark stars suggest that SGRs, AXPs, and XDINs may be linked ancestrally. Finally, we discuss the possibility of a time delay before the star enters the color superconducting phase, which can be used to estimate the density at which quarks deconfine. From observations, we find this density to be of the order of five times that of nuclear saturation.

I. INTRODUCTION

The interiors of compact stars provide a naturally-occurring environment that can be used for studying the properties of ultra-dense baryonic matter. A common means of probing this environment is through direct observations of thermal emission from the surface of compact stars. By comparing these observations with theoretical models one can retrieve key information about the physical processes occurring in matter compressed to ultra-high nuclear densities [1–6].

Most attempts to model thermal emission make use of what is called the minimal cooling scenario, which involves cooling through the minimum set of particle processes that are necessary to explain the thermal evolution of the majority of compact stars. Appropriately, it is used as a benchmark for observations of cooling neutron stars. However, there are a number of compact stars (see Table I) possessing thermal emissions significantly out of agreement with the minimal cooling scenario, indicating that other processes may be occurring and need consideration. Some specific classes of compact stars that disagree with minimal cooling, and are already distinct for some of their other features, are Soft Gamma-ray Repeaters (SGRs), Anomalous X-ray Pulsars (AXPs), and X-ray Dim Isolated Neutron stars (XDINs). It is generally accepted that SGRs and AXPs are the same type of objects, and it has been speculated before that XDINs are also related [7].

Observations indicate that SGRs and AXPs are very hot objects. Heating by magnetic field decay in the crust has been suggested [8] as a possible explanation and, at certain times during the star's evolution, it has been shown to possibly be the dominant source of heating depending on the values assumed for magnetic field decay timescales [9]. However, even with the most liberal decay timescale parameters, crustal magnetic field decay does not produce enough heat to account for SGR and AXP observations over the span of their lifetimes [10]. In fact, there is no (micro-)physical model that can explain the temperature evolution of the objects in Table I. Phenomenological studies have been performed [11], which parametrize the magnetic field decay to fit observations, and conclude that "some efficient mechanism of magnetic flux expulsion from the star's core is required". Other studies proceed by introducing an artificial heat source in an internal layer [12].

In this work, we expand on the idea that magnetic field decay from compact stars causes a re-heating of such objects. As stated above, the net effect of this mechanism on the temperature of ordinary neutron stars is much too weak to accommodate the temperatures observed for SGRs, AXPs, and XDINs. The situation changes dramatically if one assumes that these objects are made of superconducting quark matter rather than confined hadronic matter. As demonstrated in our preliminary work [13] and expanded upon herein, magnetic flux expulsion from the cores of such stars provides a very efficient and robust mechanism that can re-heat compact stars to the temperature regime observed for AXPs, SGRs, and XDINs.

II. THE VORTEX EXPULSION MECHANISM

The feature that quark matter forms a (single species) color superconductor is critical in our study. The condensation pattern that is considered here is the color-flavor-

TABLE I: Compact stars with reported thermal emissions significantly higher than can be predicted by the minimal cooling scenario, specifically SGRs, AXPs, and XDINs [9]. The listed ages are the spin-down ages modified by the vortex expulsion process ($P/3\dot{P}$), instead of the usually assumed spin-down age ($P/2\dot{P}$).

Name	$T \times 10^6$ (K)	Age (10^3 years)
SGR 1806-20	$7.56^{+0.8}_{-0.7}$	0.15
1E 1048.1-5937	$7.22^{+0.13}_{-0.07}$	2.5
CXO J164710.2-455216	7.07	0.5
SGR 0526-66	$6.16^{+0.07}_{-0.07}$	1.3
1RXS J170849.0-400910	$5.3^{+0.98}_{-1.23}$	6.0
1E 1841-045	$5.14^{+0.02}_{-0.02}$	3.0
SGR 1900+14	$5.06^{+0.93}_{-0.06}$	0.73
1E2259+586†	$4.78^{+0.34}_{-0.89}$	153
4U0142+615†	$4.59^{+0.92}_{-0.40}$	47
CXOU J010043.1-721134	$4.44^{+0.02}_{-0.02}$	4.5
XTE J1810-197	$7.92^{+0.22}_{-5.83}$	11.3
RX J0720.4-3125	$1.05^{+0.06}_{-0.06}$	1266
RBS 1223	$1.00^{+0.0}_{-0.0}$	974

†1E2259+586 and 4U0141+615 are not considered here. As argued in [14] these harbor an accreting ring that explains their extreme luminosities.

locked (CFL) phase [15–18] where quarks of all colors and flavors pair together to form Cooper pairs. Henceforth we refer to such stars as color-flavor-locked quark stars (CFLQS) [48]. We note that, while in this paper we have focused on the spin-zero CFL phase, other color superconducting phases may be more applicable, for instance the spin-1 color superconducting phases [19]. Because of stellar rotation, CFLQSs develop rotationally-induced vortices [20]. The cores of these rotational vortices are normal, or color-flavor *unlocked* [21]. If one were to consider the magnetic field in these cores, and incorporate boundary conditions between the cores and the bulk matter, one would find a difference between field strengths inside and outside of the cores. This difference would be responsible for creating a sufficient repulsive force between vortices allowing them to drag the magnetic field as they move outwards.

Although [22] found this difference to be small, it is sensitive to the value used for the QCD coupling constant. For example (cf. [22]; eqn. 3.4), with QCD coupling constants in the range of $g^2/4\pi \sim 0.1$ to 1, the amount of field expelled is 1% to 0.1%, for the case of an abrupt transition region[49]. As we are considering magnetar-strength magnetic fields on the order of up to $\sim 10^{16}$G at the star's surface, it would then seem reasonable that a significant amount in the center is expelled from the CFL matter into the rotational vortices. A more detailed study, including the running of the coupling constants [23], shows that significant inter-vortex forces can exist, but again are sensitive to the masses of the gauge fields. At this point the value of the QCD coupling constant is not well known, and so it is difficult to determine whether the inter-vortex force is sufficiently strong. In this paper we hypothesize that it is, and note how well our model matches observations, and leave further examination of the topic for future work. We also note that the (possibly more applicable) spin-1 color superconducting phases [24] that do completely exhibit the Meissner effect [19].

The total number of rotational vortices at any given time is $N_v = \kappa\Omega$, with κ being the vortex circulation and Ω the star's spin angular frequency. Hence, as such a star spins-down the number of vortices decrease. They do so by being forced radially outward [25] and upon reaching the surface are expelled from the star. The magnetic field, which is pinned to the vortices, is then also expelled and the subsequent magnetic reconnection leads to the production of X-rays [26]. Since the star's spin-down rate is proportional to the magnetic field strength, when vortices are expelled, along with the magnetic flux contained by them, the spin-down rate also decreases. Thus, the spin-down rate and magnetic field strength of a CFLQS become entirely coupled. While, this type of model has been proposed in neutron stars [27], in that setting, interactions between proton and neutron vortices prevent the clean coupling between magnetic field strength and rotation period. In the CFLQS setting there is only one type of vortex. Moreover, the presence of the crust on neutron stars further inhibits any clean expulsion of the interior magnetic field. In contrast, the vortex expulsion process from a CFLQS is very efficient at releasing X-ray photons from the stellar surface, as it possesses little or no crustal material. As such, the magnetic field is readily expelled from the star's interior where it is then able to decay through reconnection.

The X-ray luminosity from this vortex expulsion process is given by [28]

$$L_{\rm X} \simeq 2.01 \times 10^{34}\ {\rm erg\ s^{-1}}\ \eta_{{\rm X},0.1}\ \dot{P}^2_{-11}\ , \qquad (1)$$

where $\eta_{{\rm X},0.1}$ is the reconnection efficiency parameter in units of 0.1 and $\dot{P}$ the spin-down rate in units of 10^{-11} s s^{-1}. An estimate for the latter as well as the magnetic field evolution, both derived from vortex expulsion, is

$$\dot{P} = \frac{P_0}{3\tau}\left(1+\frac{t}{\tau}\right)^{-2/3}, B = B_0\left(1+\frac{t}{\tau}\right)^{-1/6}\ , \qquad (2)$$

where P_0 denotes the rotational period of the star at birth, B_0 the magnetic field at birth, and τ is a relaxation time given by

$$\tau = 5\times 10^4 \left(\frac{10^{14}{\rm G}}{B_0}\right)^2 \left(\frac{P_0}{5{\rm s}}\right)^2 \left(\frac{M}{M_\odot}\right)\left(\frac{10{\rm km}}{R}\right)^4 {\rm yrs}\ . \qquad (3)$$

One can see from equations (1 & 2) that the X-ray emission decreases as the star spins down. This X-ray emission is produced on the surface of the CFLQS, which alters its thermal evolution.

III. THERMAL EVOLUTION

To study this numerically, the general relativistic equations of energy balance and thermal energy transport need to be solved. These equations are given by ($G = c = 1$)

$$\frac{\partial(le^{2\phi})}{\partial m} = -\frac{1}{\rho\sqrt{1-2m/r}}\left(\epsilon_\nu e^{2\phi} + c_v \frac{\partial(Te^{\phi})}{\partial t}\right), \quad (4)$$

$$\frac{\partial(Te^{\phi})}{\partial m} = -\frac{(le^{\phi})}{16\pi^2 r^4 \kappa\rho\sqrt{1-2m/r}}, \quad (5)$$

respectively [2]. Here, r is the distance from the center of the star, $m(r)$ is the mass, $\rho(r)$ is the energy density, $T(r,t)$ is the temperature, $l(r,t)$ is the luminosity, $\phi(r)$ is the gravitational potential, $\epsilon_\nu(r,T)$ is the neutrino emissivity, $c_v(r,T)$ is the specific heat, and $\kappa(r,T)$ is the thermal conductivity [2]. The boundary conditions of (4) and (5) are determined by the luminosity at the stellar center and at the surface. The luminosity vanishes at the stellar center since there is no heat flux there. At the surface, the luminosity is defined by the relationship between the mantle temperature and the temperature outside of the star [29].

Heating/cooling mechanisms used in our work, other than those included in the minimal cooling scenario, are the quark direct Urca process, emissivity of which is on the order of 10^{26} erg s^{-1} cm^{-3}, and the quark modified Urca and Bremsstrahlung, which are of order 10^{19} erg s^{-1} cm^{-3}. Due to quark pairing in the CFL state, however, the direct Urca process is suppressed by a factor $e^{-\Delta/T}$ and the modified Urca and Bremsstrahlung by a factor $e^{-2\Delta/T}$ for $T \leq T_c$, where Δ is the gap parameter for the CFL phase and T_c is the critical temperature below which strange matter undergoes a phase transition into CFL matter. In the case of color-flavor and color-spin locking it was shown that the critical temperature, where the condensate melts, deviates from the BCS behavior. In the CFL case, the transition temperature is a factor $2^{1/3}$ larger than the one would expect from BCS theory ([30]). Here, we assume the validity of the BCS relation for the gap $\Delta = \Delta_0 \times \sqrt{1-(T/T_c)^2}$, with the critical temperature given by $T_c \sim 0.57\Delta_0$; Δ_0 is the magnitude of the zero-temperature gap at the Fermi surface. We point out that, because vortex expulsion becomes dominant at relatively early times, when it is included in the calculations the exact value of the critical temperature has a negligle effect on the long-term temperature evolution.

In this work the massless Goldstone bosons, due to the breaking of baryon number, were not included. In the work by [31] it was shown that at later stages in the thermal evolution these may become dominant. While [32] also confirm this, both studies look at only neutrino emission channels. Further studies including photon and neutrino emission channels [33, 34] conclude that the cooling time of CFL stars is similar to that of ordinary neutron stars. As such, we expect that the presence of massless Goldstone bosons would not significantly change the size of the shaded band in figure 1, nor our conclusions.

The heat produced by vortex expulsion occurs in an emission region just above the star's surface. In this region, the energy released by the magnetic field decaying after it has been expelled from the star's interior, is deposited. An emissivity for this process can be calculated from the energy per unit time (cf. Eq. 1) per unit volume, where the volume of interest is a shell surrounding the star. The width of the shell is estimated to be the minimum length of a vortex still inside the star, just before it is finally expelled. Such a vortex would be a distance of approximately the inter-vortex spacing away from the surface of the star. The shell width is then

$$\Delta R = 8.42\times10^{-3}\mathrm{km}\left(\frac{P}{1\mathrm{s}}\right)^{1/4}\left(\frac{R}{10\mathrm{km}}\right)^{1/2}\left(\frac{300\mathrm{MeV}}{\mu/3}\right)^{1/4}, \quad (6)$$

where μ is the average chemical potential throughout the star. In equation (6) it can be seen that the thickness of the heating layer depends weakly on the star's spin-period and density. This implies that the heating layer due to vortex expulsion will not vary significantly from one star to another.

IV. RESULTS

We have solved equations (4) and (5) numerically for models whose structure (composition and bulk properties) were computed from the Tolman-Oppenheimer-Volkoff equations [1, 2]. The equation of state used for CFL matter was the MIT bag model with massive strange quarks (cf. for example eqn. 20 in [35]). The parameters used for the equation of state are $m_s = 150$ MeV, $B^{1/4} = 145$ MeV, and Δ as described above. The results are shown in Fig. 1, where the redshifted surface temperature evolutions for various types of cooling scenarios are plotted. These scenarios are CFLQSs (CFL stars with superconductivity and the resulting vortex expulsion), and in the shaded region in Fig. 1, CFL stars (with vortex expulsion intentionally left out), *uds* stars (strange quark stars without pairing), and neutron stars [2]. The CFLQS birth spin-periods and magnetic field strengths were chosen such that the currently observed SGR/AXP/XDIN values of spin-period, spin-down rate, magnetic field strength, and luminosity would be consistent with the values derived from vortex expulsion (cf. Eq. 2). In other words by constraining two parameters in our model with observations, the spin-period, spin-down rate, magnetic field strength, and luminosity all become self-consistent. The observed data is taken from Table I, where the ages and temperatures of SGRs, AXPs, and XDINs are listed.

Figure 1 shows that CFL quark stars without vortex expulsion cool down too rapidly to agree with observed SGR/AXP/XDIN data. One might expect that the suppression processes that contribute to cooling, due to CFL

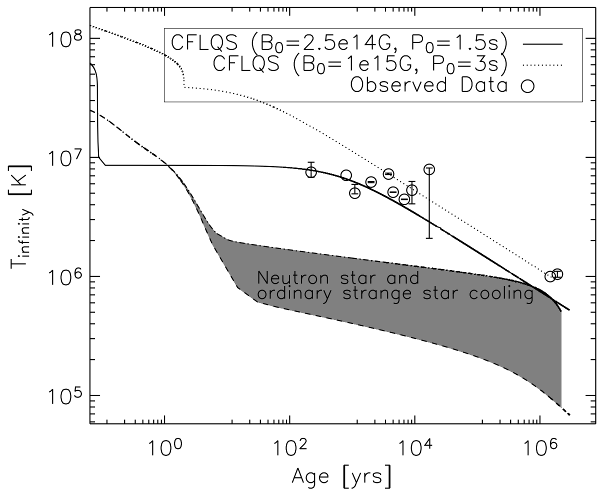

FIG. 1: Redshifted surface temperature evolution for CFLQSs (CFL stars with superconductivity and the resulting vortex expulsion). The initial spin-periods and magnetic field strengths are parameters and were chosen such that (in our model) the evolution of spin-period, spin-down rate, magnetic field strength, and luminosity would be consistent with observations. The shaded region indicates calculations of cooling scenarios for various types of neutron stars, *uds* stars (strange quark stars without pairing), and CFL stars with vortex expulsion intentionally left out. All calculations were done with stellar masses of 1.4 $M_{\odot}$ and radii of 10.5 km. The observed data is listed in Table I.

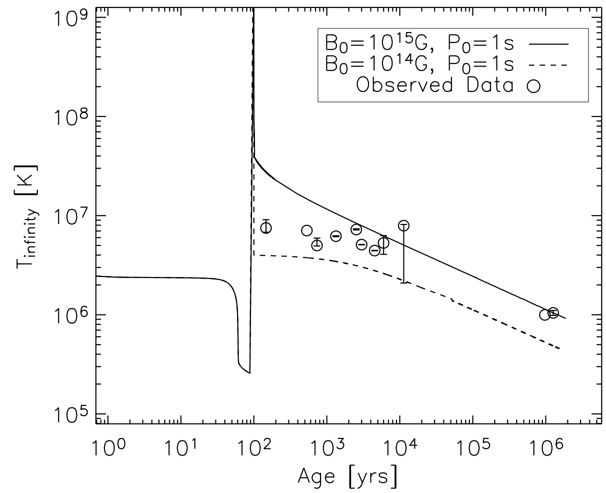

FIG. 2: Thermal evolution of a neutron star undergoing a phase transition into a CFLQS after a delay. This delay is estimated to be the time needed for the central density of a neutron star to reach the critical value at which CFL matter is favored ($\sim$ 100 years). Values of the birth temperature for the resultant CFLQS are of order $10^{11}K$, and were estimated using the quark-nova model [40]. The CFLQS birth periods and magnetic fields are as indicated.

pairing, would keep these stars hotter for longer, but pairing also changes the specific heat capacity, resulting in enhanced cooling. Standard neutron star cooling processes also lead to stars with temperatures much lower (albeit warmer than *uds* and CFL stars without vortex expulsion) than values observed for SGRs/AXPs/XDINs.

Emission due to vortex expulsion dominates all other processes except during the first few minutes. However, we also considered the possibility that a neutron star undergoes a phase transition to a CFLQS after a delay. Observational motivations for considering a delay are; (i) superluminous supernovae and hypernovae [36]; (ii) the large discrepancy between SGR/AXP ages are their progenitor supernova remnants [37]. Physically, a delay may be necessary if one considers the following; i) the time needed for a newly born compact object to spin-down sufficiently such that the center reaches nuclear densities [38], ii) the time for the temperature to reach the superconducting critical value (T_c) and the resulting vortex lattice to form, iii) the nucleation time for strangelets to start fusing together [39].

Using the quark-nova model [40] the star is reheated to roughly 10^{11} K following the transition to CFL matter. This energy is from the release of gravitational energy during the collapse as well as the latent heat released when converting from hadronic to stable strange quark matter [34, 40, 41]. Within the framework of the quark-nova scenario the neutron stars with typical values for spin-period and magnetic field, and masses greater than 1.5 solar masses, are most likely to make the transition to quark deconfinement and the subsequent color-superconducting phase. Upon this transition the magnetic field may be amplified via color ferromagnetism to roughly 10^{15} G [42], although there remains the possibility of strong fields from flux freezing during gravitational collapse and/or a dynamo at stellar birth. We choose initial values for the CFLQS's magnetic field strength accordingly. The results are shown in Fig. 2 for CFLQSs born after a delay of $\tau_{\rm QN} = 100$ years.

V. CONCLUSIONS

We have computed the thermal evolution for various types of strange quark stars and found that CFL quark stars, possessing a rotational-vortex lattice, are in good agreement with observed temperatures of SGRs, AXPs, and XDINs. Our model is applicable to any star made of a three-flavor color superconducting phase that exhibits at least a partial Meissner effect (eg. spin-0 [22] or spin-1 [19]). In our model, the CFL star spins-down as a result of magnetic braking and expels vortices, which contain magnetic flux, thus decreasing the magnetic field strength in the magnetosphere, resulting in a lower spin-down rate. Hence, the magnetic field strength and spin period become coupled. By including emission from vortex expulsion in our relativistic cooling calculations,

we have found that the unusually high temperatures of SGRs, AXPs, and XDINs can be predicted.

Other distinct signitures of CFL matter in stars include a photon fireball [43], which is of importance to explosive astophysics. However, it is only relevant during the earliest (ie. birth) stages of a CFLQS. For long-term cooling our findings indicate that vortex expulsion is the dominate emission mechanism.

From previous papers [13, 28] we have also shown that by using our model for a CFL quark star the evolution of the spin-period, spin-down rate, and magnetic field strength can also be predicted for SGRs, AXPs, and XDINs. The long-term evolution of these properties suggests an ancestral linkage between SGRs, AXPs, and XDINs. We also note a paper [44], which confirms that the birth statistics of SGRs and AXPs are consistent with the number of observed XDINs. Finally, this study suggests that a delay time between supernova and quark-nova events ($\tau_{\rm QN}$) is possible. If the estimated delay of 100 years is correct, then the density at which quark matter deconfines can be calculated to be roughly five times the nuclear saturation density [38]. The quark-nova delay can be measured by considering the discrepancy between SGR/AXP ages are their progenitor supernova remnants. Provided one had accurate measurements of this age difference, a more precise value for the density at which quark matter deconfines could be inferred.

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[48] Other condensation patterns for quark matter fail to produce enough re-heating [20, 45, 46] while pairing and superconductivity in neutron matter is also insufficient [47].

[49] We note that in the work of [22] the size of the high-density (CFL) region was assumed to be as large as the star, such that it is much greater than the screening distance. For the results of their study to be appropriate to our model, one should consider a CFL region of a size on the order of the inter-vortex seperation length.